

General large deviation principle and perturbed solutions of ODEs

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Abstract

In this seminar activity we will understand how the deterministic solution X_t of the Cauchy problem

$$\begin{cases} dX_t = b(X_t) dt & t \in [0, 1] \\ X_0 = 0 \end{cases}$$

is related to the stochastic process $(X_t^\epsilon)_{\epsilon>0}$ which is solution of the SDE Cauchy problem

$$\begin{cases} dX_t^\epsilon = b(X_t^\epsilon) dt + \sqrt{\epsilon} dB_t & t \in [0, 1] \\ X_0^\epsilon = 0 \end{cases}$$

as the parameter ϵ vanishes, using tools from the large deviation theory.

First of all we will analyze the simple case in which $b = 1$ through simulations and then also analytically, deriving a kind of statement of large deviation principle.

After that, we will introduce some tools from the large deviation theory, that is the general large deviation principle (which indeed generalizes the estimate obtained previously), the Contraction principle and Schilder's theorem.

Finally, in this generalized setting, we will be able to state and prove Freidlin-Wentzell's theorem, which will allow us to say that under the assumption of $b : \mathbb{R} \rightarrow \mathbb{R}$ Lipschitz, as $\epsilon \rightarrow 0^+$, we have that $X_t^\epsilon \xrightarrow{\mathbb{P}} X_t$ exponentially fast.

References

- [1] Dembo, A., & Zeitouni, O. (2009). *Large deviations techniques and applications* (Vol. 38). Springer Science & Business Media.
- [2] Chiarini, A., & van der Hoorn, P. (2020). *An introduction to Extreme value theory and Large deviations*. Lecture notes